

Quiz 1 Answers

$$1. y = \frac{x^3 - x^{0.7}}{\sqrt{x}} = x^{5/2} - x^{0.2}$$

$$y' = \frac{5}{2}x^{3/2} - 0.2x^{-0.8}$$

$$2. \int 3x^4 e^{x^5} dx = \int \frac{3}{5} e^u du = \frac{3}{5} e^u + C$$

$$u = x^5$$

$$= \frac{3}{5} e^{x^5} + C$$

$$du = 5x^4 dx$$

$$\frac{1}{5} du = x^4 dx$$

Lesson 5 - Substitution Applications

I. Toolbox

II Examples

I. Toolbox

(1) F is diffble \Rightarrow

$$\int_a^b F'(x) dx = F(b) - F(a)$$

Net Change Thm

$$(2) \int_{\text{ave}} \text{ on } [a, b] = \frac{1}{b-a} \int_a^b f(x) dx$$

II Examples

Ex

A car is driving along a straight road with mile markers (possibly negative)

The car is located @ mile marker 10 at time $t=0$ hours. If the velocity of the car is given by

$$s(0) = 10$$

$$v(t) = t \sqrt{2t^2 + 36}$$

what is the position of the car at time t ?
 $s(t)$ hrs

$$s(t) = \int v(t) dt$$

$$\begin{aligned} s(t) &= \int t \sqrt{2t^2 + 36} dt = \int \sqrt{u} \cdot \frac{1}{4} du \\ &\quad \left. \begin{array}{l} u = 2t^2 + 36 \\ du = 4t dt \\ \frac{1}{4} du = t dt \end{array} \right\} \\ &= \int \frac{1}{4} u^{1/2} du \\ &= \frac{1}{4} \cdot \frac{2}{3} u^{3/2} + C \\ &= \frac{1}{6} (2t^2 + 36)^{3/2} + C \end{aligned}$$

$$S(0) = 10$$

$$S(t) = \frac{1}{6} (2t^2 + 36)^{3/2} + C$$

$$10 = \frac{1}{6} (2(0)^2 + 36)^{3/2} + C$$

$$10 = 36 + C$$

$$-26 = C$$

Ans: $S(t) = \frac{1}{6} (2t^2 + 36)^{3/2} - 26$

EX The volume of water in a bucket changes at a rate of

$v'(t) \rightarrow \frac{t^2}{e^{t^3}} \frac{\text{mm}^3}{\text{hr}}$ t is hours since noon

(a) If the bucket contained 20 mm^3 of water at noon, how much is in the bucket @ 1pm?
 $t=1$

$$\int_0^1 v'(t) dt = V(1) - V(0)$$

$$\int_0^1 \frac{t^2}{e^{t^3}} dt = V(1) - 20$$

$$u = t^3$$

$$t=0 \rightarrow u=0^3=0$$

$$du = 3t^2 dt$$

$$t=1 \rightarrow u=1^3=1$$

$$\frac{1}{3} du = t^2 dt$$

$$\int_0^1 \frac{1/3 du}{e^u} = V(1) - 20$$

$$\int_0^1 \frac{1}{3} e^{-u} du = V(1) - 20$$

$$w = -u \quad u = 0 \rightarrow w = -0 = 0$$

$$dw = -du \quad u = 1 \rightarrow w = -1$$

$$\begin{aligned} -dw &= du \\ (-1)dw & \\ -1 \end{aligned}$$

$$\int_0^{-1} \frac{1}{3} e^w (-1) dw = V(1) - 20$$

$$\begin{aligned} \frac{-1}{3} e^w \Big|_0^{-1} &= V(1) - 20 \\ -\frac{1}{3} e^{-1} - -\frac{1}{3} e^0 & \end{aligned}$$

$$-\frac{1}{3} e^{-1} + \frac{1}{3} e^0 = V(1) - 20$$

$$-\frac{1}{3e} + \frac{1}{3} = V(1) - 20$$

$$\text{Ans: } V(1) = 20 - \frac{1}{3e} + \frac{1}{3}$$

(b) Set up an integral to find how much water was added to the bucket between 12:15 pm and 12:20 pm.
 $t = \frac{1}{4}$ $t = \frac{1}{3}$

$$\int_{\frac{1}{4}}^{\frac{1}{3}} \frac{t^2}{e^{t^3}} dt$$